

V International Forum on Teacher Education

## Independent Work as a Means of Activation Learning and Cognitive Activity of Future Mathematics Teachers

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### Abstract

The problem of subject preparation of pre-service math teachers is considered from the perspective of modern psychological and pedagogical science. Independent learning and cognitive activity acts as a necessary condition for improving the quality of teaching elementary mathematics. The relevance of the problem study, stated in the article, is due to the increasing amount of independent work at the university, the need to move from teaching to learning and the insufficiently developed methodology of its organization.

In this regard, this article is aimed at identifying psychological and pedagogical conditions for improving the effectiveness of independent learning and cognitive activity of future teachers of mathematics. The leading methods for studying this problem were theoretical analysis of the problem coverage, questionnaire, included observation, analysis of the products of creative activity. The study involved students of the pedagogical department of the Institute of Mathematics and Mechanics named after Nikolai Lobachevsky of Kazan Federal University and graduates, whose experience is less than three years. It was revealed that the majority of future mathematics teachers has not formed an understanding of the role and need for independent activities, and, as a result, readiness and ability to organize it. Accordingly, the study reflects the results of the organization of students' independent work, based on the idea of a "flipped classroom". The program on the organization of independent activities of future mathematics teachers, presented in the article, is aimed at ensuring active and meaningful participation of everyone in educational process, at developing skills to diagnose their own cognitive need to expand and deepen knowledge, and to master time-management skills. It will help to independently find the best ways to solve educational problems and organize such work in a future professional activity.

Keywords: future mathematics teacher, pre-service math teacher, independent activity, independent work and its organization, conditions for increasing independent work efficiency.

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Published by Kazan Federal University and peer-reviewed under responsibility of IFTE-2019 (V International Forum on Teacher Education)

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## Introduction

Modern higher education is characterized by the following trends: reliance on the idea of continuous learning - the need for specialists to constantly improve their own knowledge; a change in the organization of the educational process – an increase of the amount of independent work; transition from teaching to learning – specifically to independent activities of students. The purpose of higher education is development of a creative personality capable of self-development, self-education, and innovative activity. It is necessary to transfer a student from a passive consumer of knowledge to an active creator who knows how to formulate a problem, analyze the ways to solve it, find the optimal result, and prove its accuracy.

The issue of organizing independent activities of students is also relevant because of the fact that in accordance with the current State educational standards up to 50% of study time is allocated to independent work and, often, it is spent inefficiently. Scientific works of famous Russian and foreign psychologists reveal psychological and pedagogical essence of independent work; leading motives, goals, developmental character, and conditions for the organization of independent activity.

For our research, the works of Zimnyaya (2005) and Pidkasisty (1978) are quite relevant. Zimnyaya (2005) considers independent activity of students as the result of properly organized training activity, which is a motivating aspect for its further expansion, deepening and continuing during extra-curricular time. Independent activity, as the highest type of educational activity, requires a high level of student's self-awareness, reflexivity, self-discipline, responsibility, and gives him satisfaction as a process of self-improvement.

The effectiveness of educational process is determined by the quality of education and independent cognitive activity of students. These concepts are closely related with each other. Independent activity is the main form of learning enhancing for the following reasons:

- knowledge and skills cannot be transferred from a teacher to a student as material objects. Every student acquires them with the help of independent cognitive activity;
- the process of understanding aimed at determining the essence and content of a subject should have a certain sequence: familiarization, perception, processing, awareness, acceptance;
- the process of independent activity reveals individual tendencies of students that contribute to the formation of own opinion, own convictions, attitudes, and upholding position.

Psychological and pedagogical literature describes the concept of “independent work” along with the concept of “independent activity”.

Galskova (2000) considers independent work as a means of organizing independent activities. Independent work means the ability to identify promising goals of learning and cognitive activity, determine the means of achieving them, evaluate and adjust a process and a result of learning and cognition without the direct involvement of a teacher in this process.

Likhachev (2001) characterizes students' independent work as an active and creative form, which is a system of organizing pedagogical conditions which ensure the management of students' learning activities, and is carried out without a teacher and his direct involvement.

According to Esipov (1961), independent work is activity which is performed without the direct participation of a teacher, but according to his task at a specially provided time, while students consciously seek to achieve a goal set in the task, using their efforts and expressing a result in the form of physical or mental actions.

Pidkasisty (1978) describes independent work as any activity of students organized by a teacher,

aimed at fulfilling a stated didactic objective at a special time limit: searching for knowledge, understanding it, consolidating, forming and developing skills and abilities, generalizing and systematizing knowledge.

As a didactic phenomenon, independent work is, on the one hand, a learning task, i.e. what a student has to accomplish, on the other – a form of manifestation of the corresponding activity of memory, thinking, creative imagination while students complete the learning task, which ultimately leads him to either a completely new, previously unknown knowledge, or to deepen and expand already acquired knowledge. The final goal of the formation of independent learning activities is to achieve a level of students' development when they are able to independently set a goal of the activity, to actualize own knowledge and ways of working for problem solution, to plan and adjust their actions, to correlate a result with the intended purpose, i.e., carry out educational activities independently (Pidkasisty, 1978). That is, in the opinion of the scientist, independent work can be rightfully viewed as a means of engaging students in independent cognitive activity, also as a means of logical and psychological organization, a learning tool which corresponds to a specific didactic goal and task in each specific situation of learning. In our study, we consider independent work as a means of organizing educational and cognitive activity.

### **Problem statement**

Modern students do not know how to learn independently, which is connected, firstly, with poor motivation, and as a result, with the inability to properly manage their time. It should be noted that it is difficult for students to perceive the educational material by ear during lectures, and also experience considerable difficulties when reading, taking notes on educational, scientific literature and subsequent independent comprehension of the necessary information. The included observation over the activities of the pedagogical department graduates, whose experience is less than three years, showed that they did not pay enough attention to the independent work of their school students in training mathematics, both in class and in extracurricular activities. In the classroom they use mostly reproductive methods. All tasks are solved on the board, which discourages the development of students' cognitive activity and independence. One of the requirements of the Federal Educational Standard – "discovery of new knowledge" is not met. Young teachers do not teach students how to work with textbooks, how to search and gain knowledge independently (Fazleeva, 2017). Thus, the problem of the study lies in the fact that the majority of future teachers do not have a need for independent activity, the willingness and ability to organize it, an understanding of its role. Time allotted for independent work is spent inefficiently.

### **Study issues**

The following sequence of forms of education traditionally prevails in high school when preparing future teachers: lecture, laboratory and practical classes, independent work of students, which in this case is a form of consolidating the knowledge and skills obtained during classroom lessons and, more often, is of a reproductive nature. In this case, independent activity is a form of consolidation of knowledge and skills obtained in a classroom and, more often, it has a reproductive nature.

In our study, an attempt was made to put in the forefront the independent learning and cognitive activities aimed at obtaining new knowledge. It is followed by a lecture or a laboratory-practical lesson.

In this case, student's independent activity anticipates learning a new material or a method of action, and this is important, since, according to psychologists, the benefits of analyzing the problem appear only when students have made serious efforts to solve it.

Independent student activities can be based on:

- already existing theoretical knowledge, not yet applied in a new situation;
- unfamiliar theoretical material that should be studied with the help of the literature recommended by a teacher or using possible sources of information (reference textbooks, catalogs, dictionaries, encyclopedias, and Internet sites).

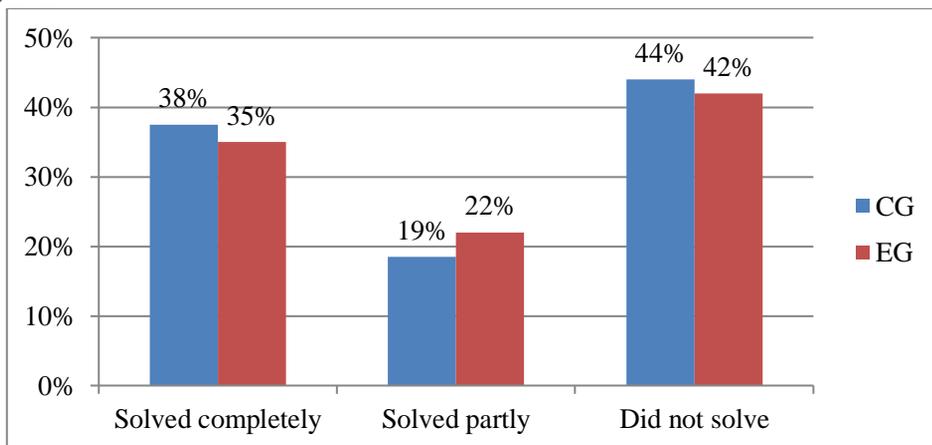
It provides:

- inclusion of students into active learning and cognitive activity;
- increasing the level of self-organization and self-control;
- setting the mind for self-education.

The organization of independent activity of students is considered on the example of the elementary mathematics section “Tasks with parameters”. To perform tasks with parameters it is not enough to apply formulas mechanically, it requires an understanding of regularities, presence of a skill to analyze a specific case based on the general properties of an object, consistency and logicity in solving, ability to combine cases into a single result. A simultaneous solution of a large set of specific equations and inequalities with the requirement of generations’ equivalence becomes possible if the level of logical thinking is sufficiently high. What is important, formation of methods for solving tasks with parameters ensures significant progress in the development of students' mathematical culture. The developmental nature of this section of elementary mathematics is determined by the fact that when solving tasks with parameters, first, a large amount of theoretical and practical material is revised, graphic culture is developed; secondly, certain thinking algorithms are evolved, as well as the ability to use verbal and graphical arguments, etc.

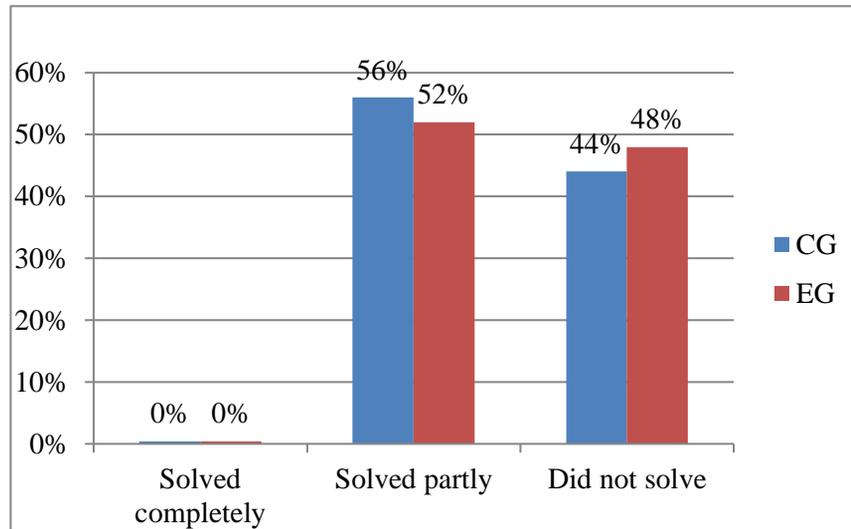
At the beginning of the course, students were tested to identify retained knowledge of the high school algebra course, on tasks with parameters. Both groups - control (CG) and experimental (EG) - were asked to analyze a quadratic equation with parameters and estimate its roots:

1. For what values of the parameter  $k$  does the equation  $(k - 1)x^2 + 2x - 1 + k = 0$  have one root?
2. For what values of the parameter  $a$  all the roots of the equation  $ax^2 - 2(a + 1)x + a - 3 = 0$  are negative?



**Fig. 1.** The results of the first task

Figures 1 and 2 show the results of solving both tasks. It should be noted that, on average, about 43% of students in both groups did not solve the first task. More than a third of the students coped with it completely (see Figure 1). At the same time, the remaining participants only equal the discriminant of the corresponding quadratic equation to zero in order for this equation to have one root, omitting the case of degeneration of the equation into a linear one when the higher coefficient vanishes.



**Fig. 2.** The results of the second task

Slightly less than a half of the students did not cope with the solution of the second task in both groups (see fig.2). The remaining students do not considered at all a case of degeneration of the equation into a linear one, but solved the task point-blank: determined the irrational roots of the equation and compared them with zero. Moreover, the resulting irrational inequalities can be solved correctly only by a few.

### **Purpose of the study**

The purpose of the study is to identify psychological and pedagogical conditions for increasing the effectiveness of independent learning and cognitive activity of future mathematics teachers.

### **Research methods**

The leading methods of the study were theoretical analysis of the problem coverage, a questionnaire, included observation, a design method, analysis of the products of creative activity. The study involved students of the pedagogical department of the Institute of Mathematics and Mechanics named after Nikolai Lobachevsky of Kazan Federal University. We organized an independent activity in two groups of students while studying the section “Tasks with parameters”. In the control group, classes were conducted in a traditional form, and in the experimental group, a “flipped classroom” method was used. After the experiment, there were control check tests of students' knowledge on the tasks with parameters.

### **Results**

Let us show how the independent activity was organized when studying the section “Tasks with parameters” in the course of elementary mathematics of students of the pedagogical department of the Institute of Mathematics and Mechanics named after Nikolai Lobachevsky. It should be noted that this section of elementary mathematics is one of the most difficult to understand and solve. This is due to the dual meaning of the very concept of “parameter”: it is a variable, which, when solving tasks, is expressed as a constant value, and it is a number which numerical value is not specified, but is assumed to be known. This knownness allows you to use a parameter as a number, but the level of freedom of action is limited by the parameter's uncertainty. Therefore, at the initial stage of exploring a parameter, it is important to use a visual-graphical interpretation of the results, which is later more than outweighed in solving complex problems.

To solve tasks with a parameter, a sufficiently high level of intelligence development is needed, since ramification of solutions is needed depending on the values taken up by a parameter. In other words, solving a task, it is necessary to classify the source equation (inequality) by the type of specific equations (inequalities), followed by a search for the final results for each type.

Let us introduce topics of the course.

1. Types of tasks with a parameter. Tasks of the first type. Linear, linear fractional, quadratic equations and inequalities with a parameter.

2. Tasks of the second type. Algebraic methods for solving tasks with parameters. Vieta theorem when solving tasks with parameters. Graphic interpretation of the quadratic trinomial.

3. Functional methods for solving tasks with parameters. Using the properties of functions.

4. Functional and graphical methods for solving tasks with parameters.

5. Geometric methods for solving tasks with parameters. Formulas for the distance between two points, distance from a point to a straight line on a flat. Equations of a line, a pair of parallel lines, intersecting lines.

6. Geometric methods for solving tasks with parameters. Inequality of a triangle, equations of a segment, a parallelogram, a circle.

7. Tasks with parameters in the Unified State Exam.

Students' independent activity in studying the section “Tasks with parameters” is organized on the basis of the idea of a “flipped classroom”, which is a learning model in which students are offered either theoretical material for independent study or practical tasks for independent solving without preliminary examination of theory. In a classroom, in the first case, a theoretical material is supported by examples, in the second – there is a collaborative search for rational ways to solve the proposed tasks with theory representation along the way.

The section “Tasks with parameters” is somewhat familiar to students, since the school course of mathematics includes consideration of the simplest equations containing parameters. Also, preparing well-performing students for the final exam in mathematics involves the analysis of fairly complex tasks with a parameter. Therefore, assuming that some students have at least a minimal amount of basic knowledge in this section, we suggest that they independently complete the tasks with parameters by known ways. During the class time under teacher's instructions, a rational way of solving a task is chosen if it was considered by some of the students during homework. If such a rational solution is still not found, a teacher himself shows it in a class.

To illustrate our work, we will focus on the study of the topics “Vieta theorem in solving tasks

with parameters” and “Graphical interpretation of a quadratic trinomial in solving tasks with parameters”.

The following tasks were offered to the students as a home task (*in a proactive mode, without preliminary examination of theory*):

1. At what values of the parameter  $a$  both roots of the equation  $(a - 2)x^2 - 2ax + a + 3 = 0$  are positive?

2. At what values of the parameter  $a$  all the roots of the equation  $ax^2 - 2(a + 1)x + a - 3 = 0$  are negative?

It should be noted that the tasks are specially chosen so that both equations have higher coefficients, depending on the parameter. But the formulation of questions in tasks implies different approaches to their solution.

During a lesson the tasks given in a proactive mode are checked.

*Task 1.* At what values of the parameter  $a$  both roots of the equation  $(a - 2)x^2 - 2ax + a + 3 = 0$  are positive?

As already noted, students begin solving the task point-blank, by calculating the roots of the equation. First they find the discriminant, which must be positive, for the equation to have two roots:  $D = 4a^2 - 4(a - 2)(a + 3) = -4a + 24 > 0$ . But they do not take into account at the same time that the roots of the equation can be equal to each other (when  $D = 0$ )

The next step is determining the roots of the equation:

$$x_{1,2} = \frac{2a \pm \sqrt{-4a + 24}}{2(a - 2)}$$

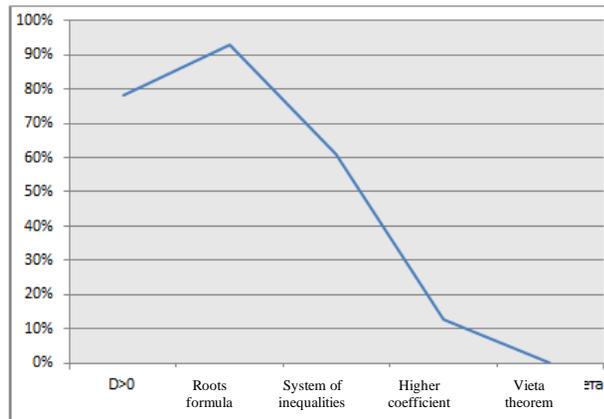
Since the roots must be positive, they get the following system of inequalities:

$$\begin{cases} D > 0, \\ x_1 > 0, \\ x_2 > 0, \end{cases} \Leftrightarrow \begin{cases} a < 6, \\ \frac{2a + \sqrt{-4a + 24}}{2(a - 2)} > 0, \\ \frac{2a - \sqrt{-4a + 24}}{2(a - 2)} > 0. \end{cases}$$

This is a system of irrational inequalities containing a parameter. It is worthy of note that the solution of such tasks gives rise to some difficulties for the majority of both school and university students. And rarely one of them solves it completely.

The general results of the task solution are as follows (see Fig. 3):

- the need of dissimilarity a higher coefficient from zero was taken into account by 13% of students;
- values of the parameter when the equation has roots (discriminant nonnegativity) were found out by 78%;
- formulas of the roots of the quadratic equation were written down by 93%;
- necessary systems of irrational inequalities were made by 61%;
- Vieta theorem was used by 0%.



**Fig. 3.** The results of solving the task in a proactive mode

Therefore, objectively, there is a need to move to a more rational solution.

As already noted, a higher coefficient of the equation depends on the parameter, and according to the statement of the task there must be two roots. Therefore, the condition of the need of dissimilarity a higher coefficient from zero ( $a - 2 \neq 0$ ) is laid down (so that the equation does not degenerate into a linear one with one root).

Further, by elaborative question, students are led to the idea of using the Vieta theorem in determining radical signs of the roots of a quadratic equation. It is worth noting that they are familiar with this material from a school course, but do not know how to apply it in a new situation.

*Vieta theorem.* If the quadratic equation  $x^2 + px + q = 0$  has roots  $x_1$  и  $x_2$ , then they satisfy the following conditions:

$$\begin{cases} x_1 + x_2 = -p, \\ x_1 \cdot x_2 = q. \end{cases}$$

In order for both roots to be positive, using the Vieta theorem, one can write out the necessary conditions in a system:

$$\begin{cases} a \leq 6, \\ x_1 \cdot x_2 = \frac{a+3}{a-2} > 0, \\ x_1 + x_2 = \frac{2a}{a-2} > 0. \end{cases}$$

It is obvious that students cope with the solution of linear fractional inequalities more successfully than with the solution of irrational ones. The solution of this system will be the set  $a \in (-\infty; -3) \cup (2; 6]$ .

*Task 2.* At what values of the parameter  $a$  all the roots of the equation  $ax^2 - 2(a+1)x + a - 3 = 0$  are negative?

At first, students independently try to solve the task using the example of the previous one, which was collaboratively analyzed.

The task question does not imply that the equation necessarily has two roots. And since the higher coefficient of the equation depends on the parameter, the solution begins with considering the case of

degeneration of the equation into a linear one with  $a = 0$ . In this case, the equation has one root,  $x = -1,5 < 0$ , which satisfies the task statement.

Next, we consider the case  $a \neq 0$  when the equation remains square and in order for it to have roots, the discriminant must be nonnegative. And in order for both roots to be negative, we must use the Vieta theorem:

$$\begin{cases} D = 4(a+1)^2 - 4a(a-3) \geq 0, \\ x_1 \cdot x_2 = \frac{a-3}{a} > 0, \\ x_1 + x_2 = \frac{2(a+1)}{a} < 0. \end{cases}$$

Solving this system of inequalities, one gets that  $a \in [-0,2; 0]$ . In response, the obtained solutions are combined  $a \in [-0,2; 0]$ .

After tasks being collaboratively analyzed, students complete a series of tasks for the independent implementation of the Vieta theorem:

1. At what value of the parameter  $p$  the equation  $x^2 + px - p^2 + 4 = 0$  has roots of different signs?
2. At what value of the parameter  $a$  the equation  $(a-1)x^2 + (2a+3)x + 2 + a = 0$  has roots of the same sign?
3. At what value of the parameter  $m$  both roots of the equation  $mx^2 + (m+1)x + m = 0$  are positive?
4. Determine the signs of roots of the equation  $ax^2 + 2(a+1)x + 2a = 0$ , without solving.
5. Determine the signs of the roots of a quadratic equation  $3ax^2 + (4-6a)x + 3(a-1) = 0$ .
6. Solve the equation  $x^2 - (2a+1)x + a + a^2 = 0$  using the Vieta theorem.

Home tasks are offered for the studied material consolidation:

1. At what values of the parameter  $a$  the equation  $x^2 - (2a+4)x - 5 - 2a = 0$  has two different, real negative roots?
2. At what values of the parameter  $a$  the equation  $x^2 - (2a-6)x + 3a + 9 = 0$  has roots of different signs?
3. At what value of the parameter  $k$  the sum of squared roots of the equation  $x^2 - (2-k)x - k - 3 = 0$  is the smallest?
4. At what value of the parameter  $a$  the equation  $x^2 + (a^2 + a - 2)x + a = 0$  has roots which sum equals 0?
5. At what value of the parameter  $a$  one of the roots of the equation  $x^2 - (3a+2)x + a^2 = 0$  nine times larger than the other?

There are also proposed the tasks *in a proactive mode*, without preliminary examination of theory:

1. At what values of the parameter  $a$  the roots of the equation  $x^2 - (2a+1)x + a^2 + a = 0$  is between the numbers 1 and 3?
2. At what values of the parameter  $a$  the roots of the equation  $(a+1)x^2 - 3ax + 4a = 0$  are larger than 1?

As in the previous case, at the beginning a teacher checks the accurateness of the task solution, similar to those solved earlier. Next, they check the tasks in a proactive mode. These tasks are chosen in such a way that there are no particular difficulties in solving the first task. Indeed, the roots of the first equation are easily determined, and choosing the necessary conditions is not difficult.

We illustrate the solution of Task 1.

The roots of the equation  $x^2 - (2a + 1)x + a^2 + a = 0$  for all valid values of the parameter are numbers  $x_1 = a + 1, x_2 = 2a$ .

After they are found, a system of corresponding inequalities is compiled:  $\begin{cases} 1 < a + 1 < 3, \\ 1 < 2a < 3, \end{cases}$  which solution  $a \in (0,5; 1,5)$  gives an answer to the task question. 60% of students cope with this task.

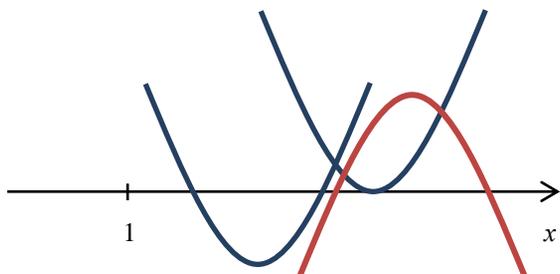
The solution of the second problem leads to the need to consider a system of irrational inequalities, which can be avoided if we use the graphical interpretation of the quadratic trinomial (Timerbaeva, 2014). It is worth noting that there is a certain number of students who already independently study the necessary theoretical material and are able to cope with the solution of the problem (about 6%).

Let us demonstrate the possible course of a lesson on the topic “Graphic interpretation of a quadratic trinomial in solving tasks with parameters”.

*Задача 2 Task 2.* At what values of the parameter  $a$  the roots of the equation  $(a + 1)x^2 - 3ax + 4a = 0$  are larger than 1?

Since the higher coefficient of the equation is parametric, they start from the case of its turning to zero. When  $a + 1 = 0$ , i.e.  $a = -1$  the equation takes the form  $3x - 4 = 0$ . Its solution  $x = \frac{4}{3}$  is greater than one. Therefore,  $a = -1$  satisfies the condition of the problem.

Let us consider the case  $a \neq -1$ . Then the equation  $f(x) = (a + 1)x^2 - 3ax + 4a$  defines a quadratic function which graph is a parabola. Let us look at the possible positions of the parabola relatively to the point  $x = 1$  of the X-axis. Since the roots of the equation must be greater than 1, the parabola intersecting or touching the X-axis to the right of  $x = 1$ .



From this we can conclude that:

- 1) the equation has roots, i.e. its discriminant is nonnegative,  $D \geq 0$ ;
- 2) the abscissa of the top of the parabola is greater than 1,  $x_0 > 1$ ;
- 3) for any direction of the parabola branches, the product of the higher coefficient and the value of the function with  $x = 1$  is positive,  $(a - 1)f(1) > 0$ .

We obtain a system of rational and linear fractional inequalities, which solution is incomparably simpler than the solution of irrational inequalities:

$$\begin{cases} a \neq -1, \\ D = 9a^2 - 16a(a + 1) \geq 0, \\ x_0 = \frac{3a}{2(a + 1)} > 1, \\ (a + 1)f(1) = (a + 1)(a + 1 - 3a + 4a) > 0. \end{cases} \Leftrightarrow \begin{cases} a \neq -1, \\ -a(7a + 16) \geq 0, \\ \frac{a - 2}{a + 1} > 0, \\ (a + 1)(2a + 1) > 0, \end{cases} \Leftrightarrow \begin{cases} a \neq -1, \\ -\frac{16}{7} \leq a < 1. \end{cases}$$

Combining the results obtained in both cases, we get the answer.

The answer.  $a \in \left[-\frac{16}{7}; 1\right)$ .

After the collaborative analyzes of tasks, students should propose a series of tasks for the independent implementation of the graphic interpretation of the quadratic trinomial:

1. At what values of the parameter the number 2 is between the roots of the equation  $x^2 + (4a + 5)x + 3 - 2a = 0$ ?
2. At what values of the parameter both roots of the equation  $x^2 + 3(a - 2)x + 2a^2 - 7a + 5 = 0$  are larger than  $-2$ ?
3. At what values of the parameter both roots of the equation  $x^2 + 4ax + 1 + 4a^2 - 2a = 0$  are less than  $-1$ ?
4. At what values of the parameter  $a$  one of the roots of the equation  $(a - 2)x^2 - 2ax + a = 0$  is greater than 3, another is less than 2?
5. At what values of the parameter  $a$  both roots of the equation  $x^2 - 3(a + 1)x + a(2a + 3) = 0$  in modulus are less than 2?

Home tasks are offered for the studied material consolidation:

1. At what values of the parameter  $a$  both roots of the equation  $x^2 + (a^2 - 1)x - a^2 = 0$  lie on the interval интервале  $(-3a; a)$ ?
2. At what values of the parameter  $k$  roots of the equation  $kx^2 - (k + 1)x + 2 = 1$  are valid and both are less than 1 in modulus?
3. At what values of the parameter  $k$  the equation  $(k - 2)x^2 + (2k - 3)x + k^2 - 3k + 2 = 0$  has two roots, one of which is less than -1 and the other is greater than -1?
4. At what values of the parameter  $a$  zeros of the function  $f(x) = x^2 - 4(a - 3)x - 20a + 35$  are located between the numbers -4 and 3?

Similarly, the process is organized during the study of the entire section "Tasks with parameters".

In the control group, training was organized in a traditional form: a lecture, laboratory and practical classes, independent work of students.

At the final stage of the experiment, testing was conducted again, where the following tasks were proposed:

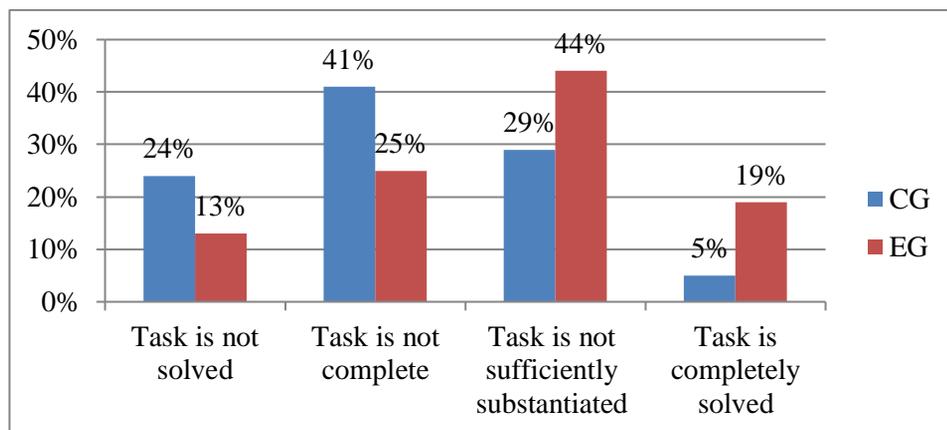
1. Find for what values of the parameter  $a$  the equation  $9^x - (3a - 10) \cdot 3^x + 2a^2 - 16a + 24 = 0$  has the only solution, larger than 1?
2. At what values of the parameter  $a$  both roots of the equation  $4x^2 + 8ax - 12x + 8a - 15 = 0$  are equal in modulus?
3. At what values of the parameter  $a$  the equation  $\frac{x^2 - (3a - 1)x + 2a^2 - 2}{x^2 - 3x - 4} = 0$  has one root?
4. Find at what values of the parameter  $a$  the system of equations  $\begin{cases} |x + 2y + 1| \leq 1, \\ (x - a)^2 + (y - 2a)^2 = 2 + a \end{cases}$  has the only solution?
5. Find at what values of the parameter  $a$  the system of equations  $\begin{cases} \sqrt{x^2 + (y - 6)^2} + \sqrt{(x - 6)^2 + y^2} = 6\sqrt{2}, \\ (x + a - 6)^2 + (y - a)^2 = 18. \end{cases}$  has only two solutions.

The solution was evaluated by the following criteria:

Content of the evaluation criterion	Gra
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	des
The task is completely solved.	5
With the correct reasoning, accurate parameter values were obtained, but the solution is not sufficiently substantiated or not all cases are considered.	4
The task is reduced to the study of: quadratic trinomial, corresponding function (exponential, logarithmic, etc.), relative position of the circles with each other, with a segment, with a region on the reference frame, etc., and is not completed.	3
The solution does not meet any of the listed criteria.	2

Let us provide with the test results.



**Fig. 4.** Experiment results

As it can be seen from the diagram, the quality of knowledge in the experimental group is higher than in the control group.

Thus, active and meaningful participation of each student in the educational process is ensured, the ability to independently find the best ways to solve problems and teamwork skills are formed.

The application of the idea of a “flipped classroom” had an impact on the development of the cognitive activity and independence of students, on the speed of new material mastering.

### Conclusions

The study showed that the organization of independent educational and cognitive activity of students activates the development of their thinking, cognitive and creative abilities.

The program for the organization of independent activities should include:

- diagnosing student's own cognitive need to expand, deepen knowledge obtained in higher education;
- definition of their own mental, personal and physical abilities;
- objective assessment of time management;
- independent selection of the study object and rational substantiation of the choice;
- setting short-term and long-term goals and developing a specific plan and program for

independent work;

- determination of the form and time of self-control.

Systematically carried independent activity based on the idea of a "flipped classroom" helps students to gain deeper and more solid knowledge. With such an organization, the following conditions are necessary:

- the goals of independent activities were meaningful and understandable for students;
- the time interval between the receipt of the task, its implementation and analyzes in the classroom should be minimized so as not to lose relevance for a student.

The professional tasks of teachers are changing – they should provide consolidating and deepening students' knowledge, acquired independently. It is necessary to reframe the curriculum, determining what will be studied independently at home, and what will be considered in class. It is necessary to develop tests for control and a system for evaluating home independent work and collective work in a classroom.

### **Acknowledgements**

The work is performed according to the Russian Government Program of Competitive Growth of Kazan Federal University.

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