

Textbook: Focus on Students' National Identity

Math textbook in teaching a modern schoolchild

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Abstract

Digitalization of education affects components of the system of teaching mathematics and the links between its individual components (aims, content, methods, forms and means of teaching). Modern students perceive and process information differently and actively use different available sources of information. A math textbook is a traditional teaching tool, and not the only one, it is also an important source of information for students. High requirements are imposed on the textbook as it should serve as a model for presenting educational material and creating conditions for students' autonomy. Students' reading proficiency is also influenced by appropriate content of a textbook and the use of effective instructional techniques. There is an academic interest to study the role and place of math textbooks in the context of digitalization of education. Our research is concerned with the ways of using a textbook in teaching mathematics at school. The article illustrates the results of the 2020 survey of student teachers of mathematics about their experiences with using mathematics textbooks during their school years. The analysis of the survey answers allowed us to identify the demand for math textbooks by modern school students, deficits and problems in methodology of teaching mathematics. The results of the research will allow us to find answers to the questions "What should a math textbook for modern pupils be like?" and "What techniques of work with a math textbook should pupils know?". The results of the research will be taken into account in the creation of methodological recommendations for math teachers to improve effectiveness of teaching mathematics.

Keywords: : mathematics education, math textbook, educational tool.

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Introduction

Literature analysis

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Russian textbooks differ from the foreign ones in several respects, according to Malaty G. (Malaty G.,1998): Russian textbooks did not undergo drastic changes; were written by prominent mathematicians and teachers, and were focused on a specific section of mathematics.

Didactic ideas about the content of education were formulated in Russia in the last 50 years, and on their basis the theories of construction of textbooks, including mathematics textbooks, were developed (Lerner I. Ya, Monakhov V. M, Kholodnaya M. A, etc.).

A mathematics textbook specifies the content of mathematics education in accordance with the programme and the requirements of the Federal State Educational Standard (FSES). The publication of textbooks for schools is funded from the federal budget of the Russian Federation. A math textbook is given to a pupil free of charge at any school in Russia.

A textbook is a book for a student. A textbook provides scientific information about different areas of mathematics. The material is presented in a format which is accessible for students and conforms with the chosen teaching methodology of the author. In this way, through its structure and educational texts, a mathematic textbook serves as a means of teaching mathematics and facilitating the development of not only mathematical knowledge and skills, but also the general ability to understand mathematical texts, to find necessary information, to perform self-monitoring. Thus, a mathematics textbook serves as a source of knowledge for a student and helps to organize their studies at school and individually.

Current regulations

In Russia, the Ministry of Education of the Russian Federation has adopted a procedure for approving the Federal List of Textbooks for Schools. Textbooks included in the Federal list of textbooks undergo several expert reviews, which consist in analyzing and assessing the textbook content according to a number of criteria.

Schools have a choice not only of a mathematics textbook from the Federal List, but also the form of such a textbook - print or electronic. In total, the Federal List of Textbooks adopted in 2020 for the next five years contains:

- 8 mathematics textbooks for grades 5-6;
- 10 geometry textbooks for grades 7-9;

- 14 algebra textbooks for grades 7-9;

- 24 maths textbooks (in algebra and the beginnings of mathematical analysis and/or geometry) for grades 10-11;

In addition to these textbooks, there are 3 more mathematics textbooks (visual geometry) for grades 5-6 that are approved for use in the educational process, as well as several special mathematics textbooks for children with intellectual disabilities.

Relevance of the research problem

The First International Conference on Mathematics Textbooks Research and Development (ICMT) in 2014 and the Second ICMT in 2017 were devoted to the study of learning tools and resources (school mathematics textbooks, teacher's manuals, learning and assessment materials, and digital and online resources). The published proceedings of these conferences (Fan L. et al., 2018; Rezat S. et al., 2019) provide a comparative analysis of the content of textbooks in several countries, suggesting that national contexts are underrepresented in the international research literature.

The results of Pisa-2018 international monitoring study (Schleicher A. 2019) show a decline in the average level of reading literacy of Russian 15-year-old students. The ability to comprehend and evaluate textual information is built by working with studying materials, primarily those presented in the textbook. According to the results of the Pisa-2018 study, improving reading literacy "requires a different information environment at school, a different structure of studying materials and new ways of working with them" (Brief results of the PISA-2018 study, 2020, p. 6). In order to change it, it is prerequisite to examine the ways in which the mathematics textbook is currently being used.

Significant scientific and pedagogical efforts, as well as those of education authorities, have been devoted to putting a textbook as a quality product on the desks of students. But, how is a math textbook used in the actual educational process in the classroom? Are all features of the textbook used in mathematics learning? Is it possible to achieve learning outcomes using a math textbook? We believe that answers to these questions will make it possible to understand the role and place of mathematics textbooks in today's teaching of mathematics and to formulate guidelines to improve the use of mathematics textbook in the learning process.

Purpose and objectives of the study

The aim of this study was to investigate the use of a textbook in teaching mathematics in Russian schools.

The objectives of the study were the following:

- To identify the main sources of information used by students when learning mathematics;

- To investigate the frequency of using mathematics textbooks in the process of learning;

- To describe the types of students' activities using a textbook in learning mathematics;

- To investigate a teacher's role in familiarizing pupils with reading mathematical sources;

- To analyze the findings.

Literature review

Research studies on the theory of mathematical education have addressed the requirements for texts, systems of exercises, and the structure of a math textbook.

The literature reflects the thesis of the overriding role of a math textbook among other sources of information for students (Popova, 2007; Gandhi H., Dewan H.K.& Ahuja A., 2018). In our view, this thesis has lost its relevance at present. A math textbook is one of the sources of information, which is very important for students.

Lerner I. Ya. (Lerner I. Ya., 1981) described the requirement for the textbook as a reflection of the structure of the educational process, all stages and components of the acquisition process of educational content. Modern math textbooks fulfil this requirement. Each paragraph in a mathematics textbook includes a problem statement, presentation of new information, highlighting the main point, presenting examples, tasks of different levels for independent solutions, and there may also be questions requiring independent answers from students.

The development of I.J. Lerner's theory is presented in the works of his followers. For example, the work of Osmolovskaya I. M. (Osmolovskaya I. M., 2017) describes the change in the model of the learning process considering the impact of informatization of education in two directions: inclusion of new information objects in the educational content and expansion of interaction of the learner with various participants of the learning process.

A systematic study of the functions and models of a textbook construction was conducted by Gelfman E. G. & Kholodnaya M. A. (2018). They considered a textbook as a multifunctional educational book "within the framework of psychodidactic approach, according to which the form, content and design of the textbook should correspond to psychological patterns of the learning activity (taking into account mechanisms of intellectual development, specific features of extrinsic and intrinsic motivation, individual cognitive inclinations of students' and their aptitudes, manifestations of personal growth, etc.)" (p. 29). The types of educational texts they developed became the basis for the development of math textbooks.

The requirements for educational texts and task material of mathematics textbooks are described by Gandhi H., Dewan H.K.& Ahuja A. (2018, pp. 55): "The emphasis in the designing of the material should be on using a language that the child can and would be expected to understand herself and would be required to work upon in a group... The entire material should be immersed in and emerge from the amalgamation of three contexts related to the child - social, cognitive and mathematical". They argue that special educational texts in the textbook should help teachers create classroom environment that generates problems, initiates discussions, provides space for explaining mathematical ideas, processes information, and develops mathematical thinking.

Researchers agree that the teacher has the primary role in fostering the students' ability to read mathematics textbooks autonomously. It is impossible not to agree with the opinion of Karp A. & Zvavich L. (2011, p. 9) that "helping students develop the skill of working with a book is one of the teacher's goals. Students rarely develop this skill on their own; for this reason, it is desirable for teachers to create conditions in which students will need to call upon this skill, and teachers will be able to demonstrate how to work with a book". Understanding the importance of such activities by teachers does not guarantee that such activities take place in the actual learning process at school. They themselves suggest that such activities are only possible with students who are interested in learning mathematics.

The influence of digitalization on all components of the methodological system of teaching mathematics, including teaching tools, which has been actively studied in recent years (Monakhov V. M., Tikhomirov S. A., & Troshina T. L., 2018; Kochagina M. N., 2019, 2020 and others). The emergence of electronic forms of mathematics textbooks (since 2015), as well as extensive digital content in the form of applications, websites or platforms for mathematics learning is being gradually described in works on the theory of mathematical education. The types of independent reading activities (pre-reading, textbook reading and post-reading activities) described by Popova A. (2007) should be substantially extended and supplemented with the tools of working with digital content.

Methodology

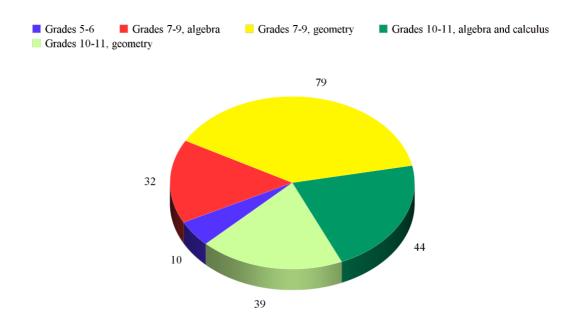
We have carried out a research on the ways of using a mathematics textbook in teaching mathematics in grades 5-11.

Experimental background of the study. The study involved 204 school graduates, who are currently Bachelor's or Master's students of two pedagogical universities in Moscow, training in "Pedagogical education" to be qualified in the field of teaching mathematics. The percentage distribution of the participants of the study is the following: 46% - 1st year Bachelor's students, 34% - 2nd year Bachelor's students, 14% - 4th year Bachelor's and 6% - 1-2 year Master's students.

Research Methods and Methodology. The participants of the research were offered survey forms, which included multiple choice questions and alternative questions, formulating their own answers, explaining the chosen answers. The survey included 15 questions, which were issued taking into account the previous responses of the survey participants, i.e. the survey was branched. There was no time limit for their answers. The survey forms were created using the forms.office.com application.

Results

When answering the question "In which grades of school did you read (study) a math textbook more frequently than in others? (Fig. 1), the majority of the survey participants (38.7%) answered "when studying geometry in grades 7-9" and only 15.7% of the answers were "when studying algebra in grades 7-9".



In which grades of school did you read (study) a math textbook MORE often than in others?

Fig. 1. Results of the answers to the question "In which classes did you read (study) a math textbook MORE often than in others?

The analysis of the responses to this question enables us to make a conclusion that in math lessons in grades 7-9, pupils considerably more often familiarize themselves with the textbook content than in senior grades: 54.4% against 40.6%.

Such results can be explained by the tradition established in Russian schools to test theoretical material at geometry lessons in grades 7-9, as well as give oral quizzes in geometry in these grades. Such forms of teaching imply autonomous preparation of students with the help of a textbook, studying theoretical material. We can also assume that it was at geometry lessons in grades 7-9 that the techniques of pupils' work with the textbook were more varied and, therefore, memorized. Most likely, pupils carried out tasks on the theoretical material of the textbook during the lesson, for example, got acquainted with the content of a certain section of the textbook, the proof of a theorem and made notes or a plan.

The following questions clarified the types of activities with a math textbook during the lesson and while homework preparation.

There were a number of responses offered to choose as an answer to the question "What kind of activity with a textbook did you do in the lesson?". The following answers were received:

"read the details of the math problem" - 89%,

"looked up formulas and formulations of theorems" - 69%,

"read historical references" - 6%,

"read proofs of theorems" - 68%,

"read the content of the paragraph and made notes" - 25%,

"read the paragraph, and then answered the teacher's questions" - 40%.

Among other activities the survey participants noted the following: "checked the correctness of the oral answer", "read the textbook and retold it", "the textbook acted as a book of tasks, as the teacher used materials from various sources when explaining new material, but not from the textbook", "the textbook only supplemented the lectures read by the teacher, so it was hardly needed in the lesson", "most often they wrote out formulas and then solved problems".

The answers received and their explanations allow us to draw a conclusion about the prevalence of reproductive teaching methods in the lesson and the use of the textbook mainly as a book of tasks or reference book.

When answering the question "How often when doing HOMEWORK at school did your teacher ask you to read (study) a math textbook? (Fig. 2), the majority of the survey participants (47%) chose the answer "for every lesson", 15% of responses noted the absence of the instruction to work with the textbook during homework, the remaining 38% of the answers can be combined with the answer "studying the textbook during homework was not systematical and rarely required".

How often when doing HOMEWORK at school did your teacher ask you to read (study) a math textbook?

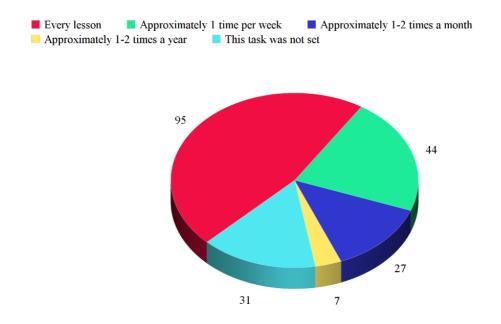


Figure 2. Results of the answers to the question "How often when doing HOMEWORK at school did your teacher ask you to read (study) a math textbook?

The question "What kind of activity with a textbook did you do when doing your homework?" offered a number of options as answers. The following answers were received:

"read the details of the math problem" " - 76%,

"searched for formulas or theorem formulations" - 65%,

"read historical references" - 5%,

"read proofs of theorems" - 39%,

"read the content of the paragraph and made notes" - 32%,

"read the content of the paragraph, and then answered the teacher's questions" - 28%.

Among other activities with the textbook, the survey participants noted the following: "read the content of the paragraph if they could not do the task or felt that they did not fully understand the topic", "checked the answers at the end of the textbook", "learned the proof of theorems as written in the textbook, followed by

retelling them in class", "prepared for exams and oral examinations". Among explanations of the types of activities with the textbook at home given by survey participants, the most frequent was "the textbook acted mainly as a task book, and if necessary, it was also used for finding definitions, formulas and theorems".

The answers received and their explanations allow us to conclude that a textbook is used for homework mainly as a task book or reference book.

When comparing the use of a math textbook at the lesson and at home, we may note a lower use of the textbook for all homework tasks, except for "reading the content of the paragraph and making an outline" (25% vs. 32%).

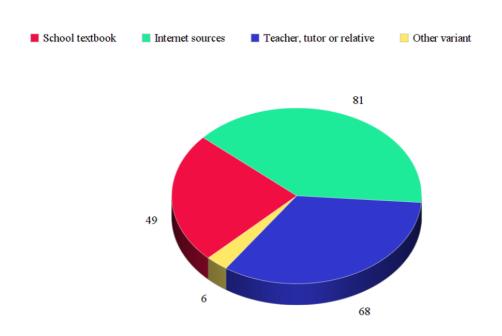
The analysis of the answers to the following question: "If you have any questions while completing math assignments, which source did you turn to more often?" made it possible to find out the reasons of the students' lower use of math textbooks for homework (Figure 3). (Fig. 3). The number of answers is the following:

"Textbook" - 24%,

"Internet sources" -40%,

"Teacher, tutor or parent" - 33%,

Other option (classmates, notes of lectures, additional literature) - 3%.



If you have any questions while completing math assignments, which source did you turn to more often?

Fig. 3. Results of the answers to the question: "If you have any questions while completing math assignments, which source did you turn to more often?"

Nowadays students are turning to digital or interactive sources of information more frequently than to a math textbook. 76% of students consider another person or Internet as a source of information. These results suggest that either pupils do not understand the necessity and appropriateness of using a textbook, or that the presentation of material in a textbook is inferior to other sources of information.

It would seem that the introduction of electronic forms of math textbooks in the Russian Federation in 2015 should have attracted students to work with a textbook, but this is not the case. To our question "Have you used the electronic form of a mathematics textbook when studying mathematics?" (Fig. 4) we got 70% of negative answers. Pupils are either not aware of their existence or they are not satisfied with the content and find other sources of information.

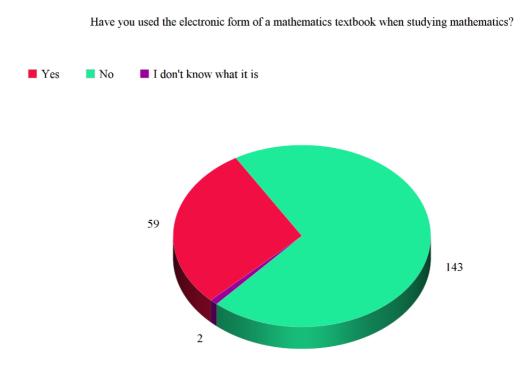


Figure 4. Results of the answers to the question "Have you used the electronic form of a mathematics textbook when studying mathematics?"

Among those respondents who did use the electronic form of a mathematics textbook, 64% used it for homework and 36% used it at the lessons.

Only 29 % of respondents, who had experience of using this form of a math textbook, noted the effectiveness of the electronic form of a mathematics textbook as compared to its paper counterpart. The most frequently cited explanations of their response were "convenience in terms of the weight of the textbook", "being able to always have access to it" rather than the way of working with the electronic form or the presentation of content. Among the respondents' explanations who had experience of using this form of a textbook when learning mathematics, but did not notice its effectiveness, the following was most frequently encountered: "I can make notes for myself in a personal textbook", "it spoils my eyesight and makes me want to leaf through a paper book", "I can look at several pages at once".

Another aim of our study was to find out how big the role of the teacher is in shaping the need to study mathematical texts. Earlier we gave the result of the question "How often did your teacher suggest you read (study) a math textbook?" (only 47% of the respondents gave the answer "every lesson", the rest indicated infrequent use of such tasks). To the question "Did your mathematics teacher suggest additional sources of

70

information other than the textbook?" (Figure 5), the response rate was 71% negative. The question "Did you read additional mathematics literature at school?" (Fig. 6), the number of negative answers was 66%. The last two questions are related; it is useful to analyze the results of the answers together. The close number of negative answers to the latter questions supports the hypothesis that math teacher has a leading role in introducing pupils to reading mathematical texts of a scientific or popular science nature. There are different ways of introducing students to supplementary reading in mathematics, but if the teacher does not use them, students very rarely show interest in learning mathematical texts on their own.

Did your teacher recommend additional sources Did you read additional sources information other than the textbook? mathematics at school? Yes Yes No No.

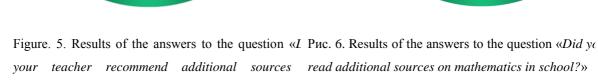
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information other than the textbook?»

The analysis of the responses to the question about additional sources recommended to pupils by their mathematics teacher revealed the main types of the following sources:

- Internet sources (educational websites for self-study, most often for the USE in mathematics, video courses, video lectures, video lessons on YouTube, Internet libraries);

- printed editions (collections of didactic materials, collections of olympiad tasks, universities' methodological textbooks for preparation for olympiad competitions and final attestation, additional textbooks, for instance, for



144

134

specialised classes).

According to the number of references to additional sources recommended by the teacher, Internet sources were mentioned by 63% of the survey participants who received such recommendations, while additionally printed sources were mentioned by 37%.

Discussions

The analysis of the survey results suggests that math textbooks are no longer the main source of knowledge and the way to organize pupils' individual work, although methodological literature has highlighted the priority of math textbooks over other sources of knowledge.

Results of the survey show that the primary use of mathematics textbook remains the same, as described in the literature nearly 15 years ago: "...the textbook acts only as a book of tasks, from which pupils take texts for self-study in class or at home" (Popova A., 2007, p. 6). Using math textbook only as a book of tasks does not allow achieving the planned learning outcomes, neither subject-specific, nor personal, nor metadisciplinary. A variety of ways of using a mathematics textbook can be obtained by relating the way of using the textbook to the six levels of Bloom's Taxonomy, as described, for example, by Heick T. (2020). The text and task material of the textbook and its structure should provide conditions for organizing different types of student's activities with the textbook.

It can also be assumed that the change in the speed and the way of comprehending information by modern pupils (generation Z) does affect the choice of the source of information (Coates J., 2006; Prensky M., 2012). Pupils are opting for quick answers to their questions rather than the kind of careful, thoughtful reading that deepens comprehension. Even electronic forms of mathematics textbooks do not appeal to students, despite the presence of interactive content. The survey confirms that the benefits of using e-textbooks in the math classroom are not obvious. Expansion of the content of e-format mathematics textbooks is worthwhile, not only in the direction of reinforcing the interactive component, but also in the direction of presenting the content that fosters reading literacy in students. Regulatory documents require that electronic forms of textbooks should not differ in design and content from the printed version of the textbooks.

Attention should be drawn to the connection between two actions: "the teacher does not organize work with the textbook systematically" - "students do not use the textbook". The first action entails the second one. Most probably one of the reasons why pupils do not want to use a math textbook, other than as a book of tasks, is the fact that teachers don't draw reasonable attention to working with the textbook. This situation

could be reversed in the situation where the teacher training curricula for mathematics teachers were to include optional and compulsory materials in mathematics, as suggested in works by (Kochagina M. N., 2020).

The choice of future math teachers in the survey was not random. Reflecting on their own experience of using a mathematics textbook, reflecting on teachers' actions, and exploring the possibilities of effective use of a mathematics textbook in teaching mathematics, in our opinion, is useful in the university training of future teachers. Our study should continue and compare the results obtained with the results of the survey of students who are less motivated to learn mathematics.

Conclusion

The use of a mathematics textbook currently in Russian schools remains within the 'knowledge-based' approach. A textbook is given the role of a reference book for finding some definition or theorem, as well as the role of a task book. In reality, educational possibilities even of those information objects that are present in the textbook are not used. Most often students work with a mathematics textbook in grades 7-9 when studying geometry, usually when learning theorems and their ready-made proofs.

Among the sources of information for learning mathematics, internet resources are more popular with students than the textbook. When learning mathematics, help from a tutor or Internet resources is used by pupils more often than reasoning and independent search for answers in a textbook. Educational sites used by pupils are mainly aimed to training preparation for the Unified State Examination in mathematics.

There is a direct correlation between teacher's recommendation of additional sources for learning mathematics and the use of such sources by pupils. More attention should be paid to the training of future teachers in techniques and methods of working with various sources of information and how to use them in teaching mathematics. The role of specially designed learning and information materials, as well as the teacher who can create conditions and organize students' activities, is crucial in the development of reading literacy of students.

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